

Top Quark Spin Correlations at the Tevatron

TIM HEAD ON BEHALF OF THE CDF AND D0 COLLABORATIONS^(*)(¹)

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Summary. — Recent measurements of the correlation between the spin of the top and the spin of the anti-top quark produced in proton anti-proton scattering at a centre of mass energy of $\sqrt{s} = 1.96$ TeV by the CDF and D0 collaborations are discussed. Using up to 4.3 fb^{-1} of data taken with the CDF and D0 detectors the spin correlation parameter C , the degree to which the spins are correlated, is measured in dileptonic and semileptonic final states. The measurements are found to be in agreement with Standard Model predictions.

1. – Introduction

Top quark physics at hadron colliders plays an important role in testing the Standard Model of particle physics and its possible extensions.

In the Standard Model the top quark has a very short lifetime, $\tau_{1/2} \approx 5 \times 10^{-25} \text{ s}$, therefore the definite spin state in which the top anti-top pair is produced is not spoiled by hadronisation effects. As a result, the direction of the spin of the top quark is reflected in the angular distributions of its decay products. In contrast to this, the spin of light quarks will flip before they decay, making the spin state they are produced in unobservable. Furthermore, the theoretical calculations necessary in order to predict the angular distributions can be performed for top pairs, resulting in precise theoretical predictions which can be tested by experiment. New physics in either the production or decay mechanism would modify these angular distributions, making spin correlations sensitive to new physics.

Until recently only one measurement of spin correlations has been performed. Using 125 pb^{-1} of data taken during Run I of the Tevatron collider at Fermilab the D0 collaboration measured a correlation coefficient in agreement with the Standard Model [1]. However, since the sample contained only six events, the sensitivity was too low to rule out the hypothesis of no spin correlations. Recently the CDF and D0 collaborations performed measurements using up to 4.3 fb^{-1} of data taken with the CDF and D0 [2] detectors, the results of which are discussed below.

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2. – Observables

In strong interactions the top and anti-top quark are produced unpolarised at hadron colliders, however the $t\bar{t}$ system is in a definite spin state. At the Tevatron about 85 %, at next to leading order, of top quark pairs are produced via quark anti-quark annihilation. At threshold these $t\bar{t}$ systems will be in a 3S_1 state, whereas the 15 % of top quark pairs produced via gluon fusion will be in a 1S_0 state. In the first case the top and anti-top quark will tend to have their spins parallel, in the second case they tend to be anti-parallel. One therefore expects to observe a correlation between the direction of the spins.

The strength of the correlation due to the production mechanism can be expressed as the asymmetry A ,

$$(1) \quad A = \frac{N_{\uparrow\uparrow} + N_{\downarrow\downarrow} - N_{\downarrow\uparrow} - N_{\uparrow\downarrow}}{N_{\uparrow\uparrow} + N_{\downarrow\downarrow} + N_{\downarrow\uparrow} + N_{\uparrow\downarrow}}$$

between the number of events with spins parallel, $N_{\uparrow\uparrow}$ and $N_{\downarrow\downarrow}$, and the number of events with spins anti-parallel, $N_{\uparrow\downarrow}$ and $N_{\downarrow\uparrow}$.

In order to measure the direction of the spin vector a quantisation axis needs to be defined. At the Tevatron three sets of quantisation axes, referred to as “spin basis”, are commonly used. They are shown in Figure 1.

The simplest is the so called “beamline basis” in which the direction of one of the incoming hadrons is used as quantisation axis. This basis is easy to construct and is optimal for $t\bar{t}$ systems produced at threshold. The production asymmetry has been calculated at next to leading order (NLO) in QCD as $A = 0.777$ [3].

The second basis is the “helicity basis” in which the momentum of the (anti)top quark in the top-anti-top quark zero momentum frame is used to quantise the (anti)top quark spin. At the Tevatron the strength of the correlation is smaller than in the “beamline basis”, in NLO QCD $A = -0.352$. The opposite sign arises due to the fact that the spins tend to be anti-parallel in this basis.

Finally the third basis is the “off-diagonal basis”. The direction of the quantisation axes are defined by the angle ω with respect to the (anti)top quark momentum. The angle ω is given by $\tan \omega = \sqrt{1 - \beta^2} \tan \theta$, where β is the speed of the top quark and θ is its scattering angle. This basis interpolates between the “beamline basis” close to threshold (low β) and the “helicity basis” above threshold (large β). The production asymmetry is $A = 0.782$. While this is slightly larger than in the “beamline basis” it is more complex to reconstruct.

The angular distribution of decay product i in the top quark rest frame is given by:

$$(2) \quad \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i} = \frac{1}{2} (1 - \alpha_i \cdot \cos\theta_i)$$

where θ_i is the angle between the direction of flight of decay product i and the direction of the spin vector; α_i is the so-called spin analysing power. From Equation 2 it is clear that the angular distribution of a decay product with $\alpha_i = 0$ contains no information about the direction of the top quark spin and the angular distribution of a decay product with $\alpha_i = \pm 1$ will contain most information. The spin analysing power of the various top quark decay products are listed in Table I. The particles with the highest spin analysing power are the lepton and the down type quark from the W boson decay.

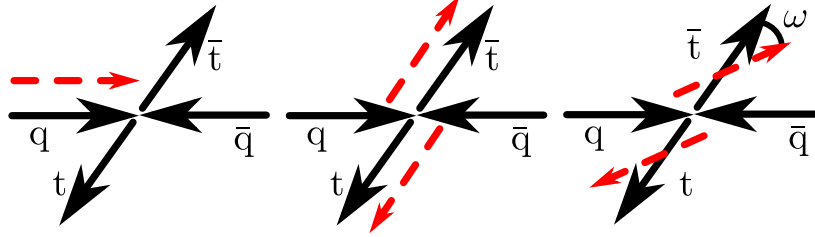


Figure 1. – The three choices of quantisation axis used at the Tevatron. The “beamline basis” (left) is optimal for top pairs produced at threshold, the “helicity basis” (centre) is used for above threshold top pairs and the “helicity basis” (right) interpolates between the two.

In order to observe a correlation between the direction of the spin of the top and anti-top quark one must consider the angle θ of a decay product of the top quark and the angle of a decay product of the anti-top quark simultaneously. The double differential distribution for a top quark decay product i and anti-top quark decay product j is given by:

$$(3) \quad \frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_i d\cos\theta_j} = \frac{1}{4} (1 - A\alpha_i\alpha_j \cos\theta_i \cos\theta_j)$$

where σ is the total cross section, A is the production asymmetry, and $\alpha_{i,j}$ is the spin analysing power of the i, j -th decay product. In all analyses presented here the spin correlation parameter $C = A\alpha_i\alpha_j$ is measured. A measurement of the distribution given in Equation 3 should be performed as follows:

1. Reconstruct the top and anti-top quark momenta in the laboratory frame,
2. Perform a boost from the laboratory frame to the rest frame of the $t\bar{t}$ system. Define the vectors \hat{b}_i and \hat{b}_j along which to quantise the top and anti-top quark spins respectively.
3. Boost the top (anti-top) quark decay product to the top (anti-top) quark rest frame and calculate $\cos\theta_{i,j} = \hat{b}_{i,j} \cdot \hat{q}_{i,j}$.

The difference between the case of no spin correlations, $A = 0$, and SM spin correlations as measured in the “beamline basis”, $A = 0.777$, using leptons as spin analysers is shown in Figure 2.

TABLE I. – Spin analysing power of the top quark decay products. The up type quark, down type quark, neutrino and lepton are the decay products of the W boson. For the antiparticles the sign is reversed.

	lepton, down type quark	neutrino	up type quark	b quark
analysing power α	+1	+0.31	+0.31	+0.41

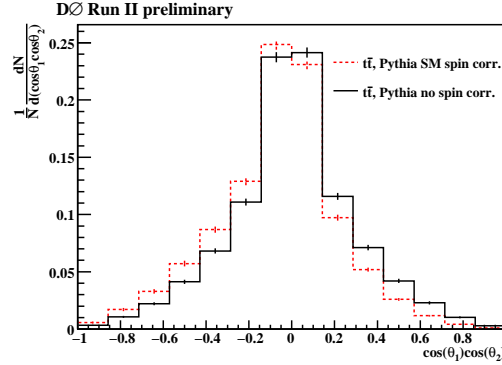


Figure 2. – The distribution $\cos \theta_1 \cos \theta_2$ for a sample of top anti-top quark events using generated partons. With spin correlations (red dashed) and without (black solid). Here both top quarks decayed to leptons which subsequently were used as spin analysers [4].

3. – Measurements

While in theory the down type quark is as powerful a spin analyser as the lepton, it is more difficult to identify in practise. This leads to two different approaches. In the first one, one selects a pure sample of top pairs in which both the top and anti-top quark decay to leptons. In the second a sample with higher statistics is selected by requiring only one top quark to decay to a lepton. In the following the advantages, challenges and results are discussed for both approaches.

3.1. Dilepton final states. – The advantages of the dilepton final state are that it is simple to identify the final state particles of interest and the high purity of the sample. The disadvantage is that one suffers from a low branching ratio and needs to deal with two neutrinos when reconstructing the kinematics of the event. Both CDF and D0 select events with two high p_t leptons of opposite charge and at least two jets. The detailed event selections are described in References [5, 4] for CDF and D0, respectively. In final states with same flavour leptons (e^+e^- and $\mu^+\mu^-$) the main background arises from Drell-Yan, $Z/\gamma^* \rightarrow \ell^-\ell^+$, production. In the $e\mu$ final state the main background is instrumental, this occurs mainly due to W +jets events in which a jet is misidentified as a lepton. The second largest background arises due to semileptonic decays of $Z/\gamma^* \rightarrow \tau^-\tau^+$. Further sources of background in all three final states are the diboson processes WW , WZ and ZZ . Both signal and background are modelled using Monte Carlo simulation, except for the instrumental background which is estimated from data.

In order to reconstruct the momentum of the top and anti-top quark, one needs to deal with the two neutrinos in the final state. To fully characterise the kinematics of the final state one needs 18 quantities, assuming the masses of the final state particles are known. While the leptons and jets are observable in the detector, the two neutrinos escape detection. It is possible to infer the sum of the momenta of the neutrinos in the x and y plane from the missing transverse energy, \cancel{E}_T^x and \cancel{E}_T^y . Using this information and making an assumption about the mass of the W boson and the top quark it is possible to write down a set of quartic equations which fully describe the final state. Solving them yields up to four solutions per event. Additionally one needs to try both lepton-jet pairings which increases the number of possible solutions to eight.

In the CDF measurement a likelihood function is constructed from several observables and maximised with respect to the unknown neutrino momenta ($\vec{p}_\nu, \vec{p}_{\bar{\nu}}$) and the energies of the bottom quark jets ($E_b^{\text{guess}}, E_{\bar{b}}^{\text{guess}}$):

$$L(\vec{p}_\nu, \vec{p}_{\bar{\nu}}, E_b^{\text{guess}}, E_{\bar{b}}^{\text{guess}}) = P(p_z^{t\bar{t}}) P(p_T^{t\bar{t}}) P(M_{t\bar{t}}) \times \\ \frac{1}{\sigma_b} \exp\left(-\frac{1}{2} \left(\frac{E_b^{\text{meas}} - E_b^{\text{guess}}}{\sigma_b}\right)^2\right) \times \frac{1}{\sigma_{\bar{b}}} \exp\left(-\frac{1}{2} \left(\frac{E_{\bar{b}}^{\text{meas}} - E_{\bar{b}}^{\text{guess}}}{\sigma_{\bar{b}}}\right)^2\right) \times \\ \frac{1}{\sigma_x^{\text{MET}}} \exp\left(-\frac{1}{2} \left(\frac{\cancel{E}_x^{\text{meas}} - \cancel{E}_x^{\text{guess}}}{\sigma_x^{\text{MET}}}\right)^2\right) \times \frac{1}{\sigma_y^{\text{MET}}} \exp\left(-\frac{1}{2} \left(\frac{\cancel{E}_y^{\text{meas}} - \cancel{E}_y^{\text{guess}}}{\sigma_y^{\text{MET}}}\right)^2\right)$$

where $P(p_z^{t\bar{t}})$, $P(p_T^{t\bar{t}})$ and $P(M_{t\bar{t}})$ are probability density functions obtained from PYTHIA $t\bar{t}$ Monte Carlo events, $E_{b,\bar{b}}^{\text{meas}}$ the measured energies of the bottom/anti-bottom quark jets, $\cancel{E}_{x,y}^{\text{meas}}$ the measured components of \cancel{E}_T , and σ_i the respective resolutions. The maximisation is performed for both lepton-jet pairings and the combination with the larger L is kept.

As the PYTHIA Monte Carlo simulation does not contain spin correlations, templates for values of $C = -1, 0.8, \dots, 1$ are obtained by reweighting the signal Monte Carlo at the generator level using a weight $w \sim 1 - C \cdot \cos\theta_1 \cos\theta_2$. For each value of C a two dimensional template in the decay angles of the lepton and anti-lepton, $\cos\theta_{\ell^+}, \cos\theta_{\ell^-}$ and a template in the decay angles of the bottom quark jets, $\cos\theta_b, \cos\theta_{\bar{b}}$ is created. The two templates are fit with an analytical function $f^{\ell,b}(x, y; C)$. The measurement is then performed on the N candidate events by maximising the likelihood function:

$$L(C) = \prod_{i=0}^N f^l(x, y; C) f^b(x, y; C).$$

In order to extract limits from the measurement, a confidence belt according to the Feldman-Cousins prescription [6] is created. This naturally includes both statistical and systematic uncertainties and allows one to decide before looking at the data whether to quote a one or two sided limit. Using 2.8fb^{-1} of data the best fit value is $C = 0.32_{-0.78}^{+0.55}(\text{stat} + \text{syst})$ and the corresponding confidence belts are shown in Figure 3. The measurement was performed in the ‘‘helicity basis’’. The result is consistent with the expected value of $C = 0.782$. The largest contributions to the systematic uncertainty come from evaluating the PDF uncertainties and the finite number of Monte Carlo events used to form the templates.

At D0 the neutrino weighting technique is used to solve for the event kinematics. By making an assumption about the rapidity, η , of the neutrino and anti-neutrino, it is possible to solve the event kinematics, while not using \cancel{E}_T^x and \cancel{E}_T^y in the process but instead to assign a weight, w , to each solution given by:

$$w = \exp\left(-\frac{(\cancel{E}_T^x - \nu_x - \bar{\nu}_x)^2}{\sigma^2}\right) \times \exp\left(-\frac{(\cancel{E}_T^y - \nu_y - \bar{\nu}_y)^2}{\sigma^2}\right)$$

where $\nu_{x,y}$ and $\bar{\nu}_{x,y}$ are the x and y components of the neutrino and anti-neutrino momentum for a given solution and σ is the \cancel{E}_T^x resolution. Many solutions are obtained by sampling the neutrino and anti-neutrino rapidity based on Monte Carlo simulation. No dependence of the neutrino rapidity on the presence of spin correlations is observed.

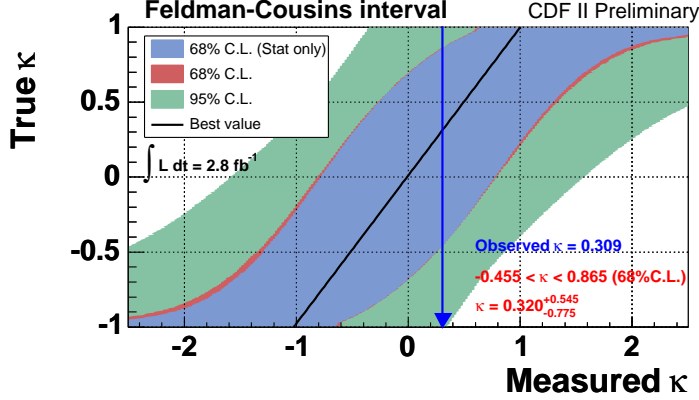


Figure 3. – The 68% (stat only), 68% and 95% Confidence Level intervals constructed according to the Feldman-Cousins prescription including statistical and all systematic uncertainties for the CDF measurement. The best fit value is $C = 0.32^{+0.55}_{-0.78}$ [5].

The weighted mean of all solutions for an event is used as estimator for the true value of $\cos \theta_{\ell^+} \cos \theta_{\ell^-}$.

As for the CDF measurement, the PYTHIA Monte Carlo simulation is used to model the signal sample. A one dimensional template in the variable $\cos \theta_{\ell^+} \cos \theta_{\ell^-}$ is created for $C = 0$ and $C = 0.777$ by reweighting the distribution at the generator level. In order to extract a value of C a linear combination of the two templates is fit to the data.

Pseudo-experiments are created for each value of C and fit with signal and background templates. Each source of systematic uncertainty is considered as a nuisance parameter during the fit. Feldman-Cousins confidence belts are constructed from the pseudo experiments. Using up to 4.2 fb^{-1} of data the best fit value is $C = -0.17^{+0.64}_{-0.53} (\text{stat} + \text{syst})$. In this measurement the “beamline basis” was used and the measured value is consistent with the Standard Model expectation of $C = 0.777$ at the two sigma confidence level.

The two main sources of systematic uncertainty are the variation of the assumed top mass during the event reconstruction from 175 GeV to 170 GeV and the test of the reweighting method. For the latter, the two PYTHIA signal templates were replaced by ALPGEN, which contains spin correlations, and MC@NLO where spin correlations were turned off.

3.2. Semileptonic final states. – Selecting semileptonic events results in a higher yield, but the challenge is to identify the down type quark. This is done probabilistically by choosing the jet closest to the bottom type jet in the W boson rest frame [7], which will result in picking the correct jet about 60% of the time.

Events are selected by requiring at least one high p_T , central lepton, large missing transverse energy and four or more jets, one of which must be identified as a b-jet. The backgrounds are estimated both from simulation and data. For details of the selection see Reference [8]. Using 4.3 fb^{-1} of data a total of 1001 events are selected of which 786 are expected to be top pair events.

When produced in pairs the top and anti-top quark either have the same helicity or

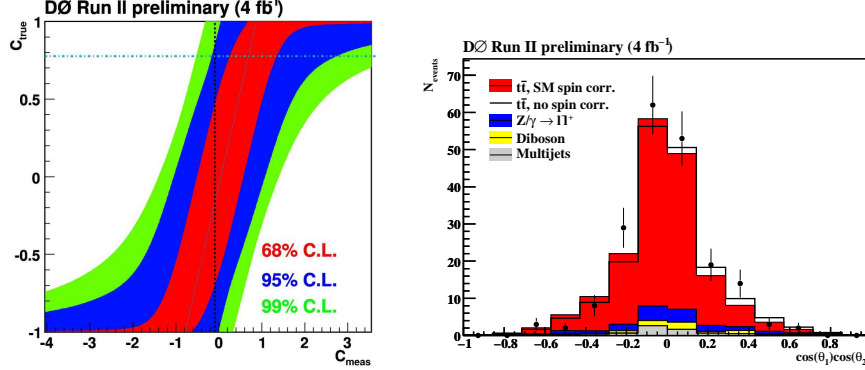


Figure 4. – Left: the 68%, 95% and 99% Feldman-Cousins confidence belts are shown. The best fit value can be read of at the intersection of the dashed black line and the thin blue line. Right: The sum of all dilepton channels is shown. The open black histogram shows the expected distribution for the case of no spin correlations, $C = 0$ and the filled red histogram the expected distribution for Standard Model spin correlations, $C = 0.777$ [4].

opposite helicity. The fraction of top pairs with opposite helicity is given by:

$$f_O = \frac{\sigma(\bar{t}_R t_L) + \sigma(\bar{t}_L t_R)}{\sigma(\bar{t}_R t_R + \bar{t}_L t_L + \bar{t}_R t_L + \bar{t}_L t_R)},$$

where $\sigma(\bar{t}_L, R t_L, R)$ denotes the cross section for each possible helicity configuration. Using Equation 1 one can show that a measurement of f_O is equivalent to a measurement of A in the helicity basis.

One template for top pairs with same helicity and one template for top pairs of opposite helicity are created using a modified version of the HERWIG event generator. The opposite helicity fraction is extracted with a binned maximum likelihood fit of the two templates to the data, with contributions from backgrounds taken into account. The best fit value is $f_O = 0.80 \pm 0.26(\text{stat} + \text{syst})$ or equivalently $A = 2f_O - 1 = 0.60 \pm 0.52(\text{stat} + \text{syst})$. This is consistent with the Standard Model expectation of $A = 0.4$. The two main systematic uncertainties are Monte Carlo statistics and jet energy scale.

4. – Conclusions

The spin correlation parameter C has been measured in dilepton and semileptonic decays of top and anti-top quark pairs using up to 4.3 fb^{-1} of data collected with the CDF and D0 detectors. Measurements were performed in the “beamline”, “helicity” and “off-diagonal” bases. The measurements are found to be in agreement with the Standard Model predictions. All three measurements are still statistically limited. Considering that the Tevatron collider has delivered nearly twice as much integrated luminosity since the analyses have been performed, updates of all measurements can be expected soon.

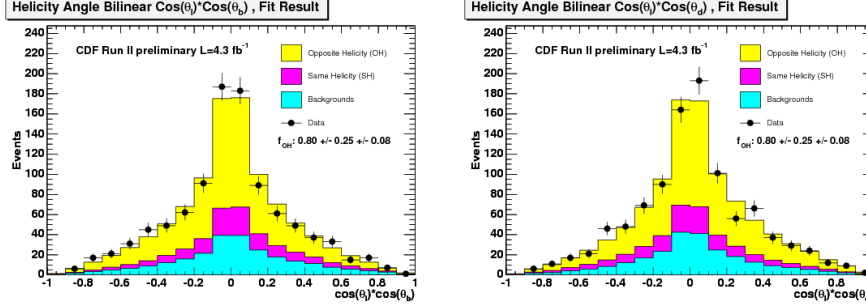


Figure 5. – The best fit of same helicity, opposite helicity and background templates for the CDF semileptonic decay channel. On the (left) the distribution of the product of the decay angle of the lepton and the bottom quark. On the (right) the distribution of the product of the decay angle of the lepton and the down type quark. The best fit value from a simultaneous fit to both distributions is $f_O = 0.8 \pm 0.26(\text{stat} + \text{syst})$ or $C = 0.6 \pm 0.52(\text{stat} + \text{syst})$ [8].

REFERENCES

- [1] ABBOTT B. *et al.*, *Phys. Rev. Lett.*, **85** (2000) 256.
- [2] ABAZOV V. M. *et al.*, *Nucl. Instrum. Meth.A*, **565** (2006) 463.
- [3] BERNREUTHER W., BRANDENBURG A., Si Z. G. and UWER P., *Nucl. Phys.B*, **690** (2004) 81.
- [4] THE D0 COLLABORATION, *Spin Correlations in $t\bar{t}$ Production in Dilepton Final States*, D0 note 5950-CONF.
- [5] THE CDF COLLABORATION, *A Measurement of $t\bar{t}$ Spin Correlations Coefficient in 2.8 fb^{-1} Dilepton Candidates*, CDF note 9824.
- [6] FELDMAN G. J. and COUSINS R. D., *Phys. Rev.D*, **57** (1998) 3873.
- [7] MAHLON G. and PARKE S. J., *Phys. Rev.D*, **53** (1996) 4886.
- [8] THE CDF COLLABORATION, *Measurement of $t\bar{t}$ Helicity Fractions and Spin Correlation Using Reconstructed Lepton+Jets Events*, CDF note 10048.